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1 Introduction

Gust (or buffeting) response is an oscillation of a bridge structure resulting from velocity fluctuations in the oncoming flow. This type of response occurs in every bridge regardless of the design of the cross-section of the deck. This is the point that is different from other aeroelastic phenomena like vortex-induced vibration and flutter. This phenomenon may not cause immediate collapse of the bridge, but may cause fatigue damage or serviceability problems. With the increase of span length, cable-stayed bridges are becoming so flexible that gust response can be critical in the design, especially at their erection stage.

Gust response is predicted by wind tunnel tests simulating atmospheric turbulence on a model of the full bridge or by gust response analysis. The effects of wind direction on the response of bridges have never attracted much attention among engineers due to the belief that bridge response examined under wind coming normal to its longitudinal axis would be the worst case and the coverage of this case should be sufficient to serve the purpose of the study. In many cases this is a reasonable assumption. However, some experimental observations[1] show that the bridge gust response can be larger due to wind with a horizontal skew angle than in wind perpendicular to the bridge axis.

Recently, a few analytical attempts have been made to investigate this problem, the first being done by Xie *et al.*[2] The analysis is carried out by introducing a concept of "effective" values of mean wind velocity, deck width, and scales of turbulence. A sample calculation using this analysis gave reasonable agreement with a set of experimental observations. However, it has been pointed out that its application should be limited to small yaw angle cases, and also that it cannot be applied to bridges with unsupported free ends without further modifications.

Kimura *et al.*[3] developed a modified gust response analysis for a cantilever beam with a flat plate cross-

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section which can be applied to large yaw angle cases by considering two effective wind velocities separately. One is the velocity component normal to the model axis and the other is the component normal to the free end of the model. The analysis gives good reproductions of the vertical gust response characteristics of the model regarding wind yaw angles.

Scanlan[4] also attempted a similar modified gust response analysis by taking the chordwise 2D strips unrelated to the flow direction. Its formulation includes not only vertical response but also horizontal and torsional. However, the analytical results have not been fully confirmed experimentally.

In the present paper, modified vertical gust response analysis[3] is applied to cable-stayed bridges in their erection stages by making some assumptions and approximations. The calculated response in yawed wind is compared with experimental results,[5,6] and the validity of these assumptions and approximations are discussed.

2 Modified Vertical Gust Response Analysis for Yawed Wind

2.1 Definition of Effective Wind Velocities

Only the aerodynamic forces acting on the bridge deck are considered. The effective wind velocity can be taken as the wind velocity component normal to the leading edge. This is the concept originally for an infinitely long plate and cannot be stricrly applied for cable-stayed bridges with finite length. However, it is assumed that this definition is applicable by correcting the slope of the lift coefficient used in the analysis as described in Section 2.2.

For cable-stayed bridges under construction there are two leading edges, one being the side and the other the end of the deck. Then corresponding effective wind velocity becomes the wind velocity component normal or parallel to the bridge longitudinal axis (**Fig. 1**). Therefore, the effective wind velocity U_e becomes

$$U_{eC} = U\cos\beta \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

$$U_{eS} = U\sin\beta \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where U is the mean wind velocity and β is the wind yaw angle taken from normal to the bridge axis. In the following analysis, the calculation is conducted separately based on either of these two effective wind velocities. The analysis based on Eq. (1) will be called the "cosine case," and that based on Eq. (2) will be called the "sine case." The subscripts C and S correspond to the cosine and sine cases, respectively.

2.2 Formulation of the Buffeting Lift Force on a Strip

The strip theory approximation is used and the strips are taken in the mean flow direction. In order to simplify the formulation, it is assumed that the buffeting force

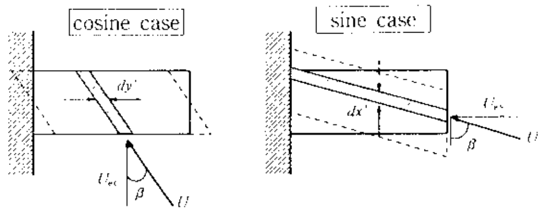

Fig. 1 Cosine and sine cases

acting on the parallelogram shown in Fig. 1 with broken lines is equal to the force on the bridge deck. All the strips considered in the analysis then have the same chord length. With the quasi-steady approximation, the buffeting lift force, dL, acting on a strip can be expressed as follows.

For the cosine case,

$$\begin{aligned} dL_C(y', t) &= \frac{1}{2}\rho U_{eC}^2 B\, dy'(C_{L\alpha C} + C_{DC})w(y', t)/U_{eC} \\ &= \frac{1}{2}\rho UB\, dy'(C_{L\alpha C} + C_{DC})w(y', t)\cos\beta \end{aligned} \cdots (3)$$

For the sine case,

$$dL_S(x', t) = \frac{1}{2}\rho Ul\, dx' C_{L\alpha S} w(y', t)\sin\beta \cdots (4)$$

y', x': spanwise and chordwise coordinate on the bridge deck
t: time
ρ: air density
B: bridge deck width
l: overall span length
dy', dx': width of the strip for the cosine and sine cases
$C_{L\alpha C}, C_{L\alpha S}$: slope of the lift force coefficient for the cosine and sine cases
C_{DC}: drag coefficient for the cosine case normalized with the deck width
w: vertical component of the fluctuating wind velocity

The contribution from the fluctuating wind velocity in the mean wind direction, u, to the buffeting lift force is neglected because the lift coefficient of the bridge deck to which this analysis is applied is close to zero when the wind comes from the horizontal direction. The contribution from the drag coefficient for the sine case is also neglected because it is difficult to estimate the coefficient and the effect would not be large.

To include the effects of the finite span length, the slope of the lift coefficient for the cosine case, $C_{L\alpha C}$, used in Eq. (3) is obtained by correcting that from the section model test. The correction is made based on the following expression, which is valid for flat plates.[7]

$$\frac{C_{L\alpha C}}{C_{L\alpha 2D}} = \left[1 + \frac{C_{L\alpha 2D}}{\pi AR}\right]^{-1} \cdots\cdots\cdots\cdots (5)$$

where $C_{L\alpha 2D}$ is the slope of the lift coefficient with the section model test and AR is an aspect ratio that is l/B for the cosine case and B/l for the sine case.

For the sine case, the slope of the lift coefficient, $C_{L\alpha S}$, is not measured during the normal wind tunnel testing procedure, and therefore it is estimated as follows. First, the ratio of $C_{L\alpha S}/C_{L\alpha C}$ for a flat plate with the same aspect ratio as the bridge deck is calculated. As the slope of the lift coefficient for flat plates with a small aspect ratio becomes larger with the angle of attack,[8] the equivalent lift slope, which may be obtained by taking the average slope between the origin and the largest expected angle of attack, is used in the computation. The ratio is assumed to be equal for the flat plate and the bridge deck with the same aspect ratio, and by multiplying $C_{L\alpha C}$ of the bridge deck from Eq. (5), $C_{L\alpha S}$ of the deck is obtained.

2.3 Formulation of the Generalized Buffeting Lift Force on the Bridge Deck

Because the amount of the vertical displacement of a single strip at an instant is not uniform, the mode shape should be taken into account to formulate the generalized force on the strip. For a flat plate, the resultant lift force acts at the quarter chord even in the turbulent flow, and the same pressure distribution is assumed for the bridge deck.

For the cosine case, it is assumed that the deck deflection is the same everywhere inside a strip and that at the quarter chord of the strip is the representative. On the other hand, the contribution of the mode shape is fully considered for the sine case. The generalized lift force of the r-th mode acting on a strip, dF, can then be expressed as follows.
For the cosine case,

$$dF_{zCr}(y', t) = dL_C(y', t) \times \phi_r\left(y' + \frac{1}{4} B \tan\beta\right) \cdots (6)$$

For the sine case,

$$dF_{zSr}(x', t) = \frac{1}{2} \rho U \frac{l}{l_s} dx' w(x', t) \sin\beta \times \int_0^{l_s} C_{L\alpha S}(s)\phi_r'(s)\, ds \cdots\cdots\cdots (7)$$

$\phi_r(y')$: r-th mode shape along the bridge span
s: coordinate along the strip
l_s: length of the strip
$C_{L\alpha S}(s)$: solpe of the lift coefficient defined as the function of s
$\phi_r'(s)$: r-th mode shape along the strip

$C_{L\alpha S}(s)$ is normalized so that

$$\frac{1}{l_s}\int_0^{l_s} C_{L\alpha S}(s)\, ds = C_{L\alpha S} \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

and the distribution is assumed to be the same as that on flat plates.

The power spectrum of the generalized buffeting lift force of the r-th mode on the entire bridge deck can be obtained by the following equations.
For the cosine case,

$$S_{FzCr}(f) = \left[\frac{1}{2} \rho UB(C_{L\alpha C} + C_{DC}) \cos\beta\right]^2 \times \int_0^l \int_0^l S_{ww}(f, y', y'')\phi(y' + e)\phi(y'' + e)\, dy'\, dy'' \cdots (9)$$

For the sine case,

$$S_{FzSr}(f) = \left[\frac{1}{2} \rho U \sin^2\beta \times \int_0^{l_s} C_{L\alpha S}(s)\phi_r'(s) ds\right]^2 \times \int_0^B \int_0^B S_{ww}(f, x', x'')\, dx'\, dx'' \cdots\cdots\cdots\cdots\cdots (10)$$

where S_{ww} is the cross spectrum of the vertical-component of the fluctuating wind velocity and $e = 0.25B \tan\beta$. The cross spectrum analytically obtained from the von Kármán spectrum[9] was used in the analysis.

2.4 Aerodynamic Damping

Because only the vertical response is considered in the present study, the contribution of the self-excited force is included by adding aerodynamic damping.

When the wind direction is normal to the bridge longitudinal axis, the aerodynamic damping for the r-th mode is estimated by the following expression based on quasi-steady approximation with correction using the Theodorsen function, which is the theoretical expression of the unsteady aerodynamic forces acting on flat plates.

$$h_{a0} = \frac{\rho BUF(k)}{4\omega_r M_r} \int_0^l (C_{L\alpha C} + C_{DC})\phi_r^2(y')\, dy' \cdots\cdots (11)$$

$F(k)$: real part of Theodorsen function
$k = 0.5\, \omega_r B/U$
ω_r: circular natural frequency of the r-th mode
M_r: generalized mass of the r-th mode

In the analysis for yawed wind, the aerodynamic damping is assumed to be proportional to $\cos\beta$ since it has been found to be approximately proportional to $\cos\beta$ for cantilever beams with flat plate cross-section in a smooth flow.[3]

2.5 Prediction of r.m.s. Response

A linear relationship holds between two-dimensional and three-dimensional aerodynamic forces if the strip theory approximation is applied. Therefore, considering the aerodynamic admittance, $|X(f)|^2$, the power spectrum of the generalized coordinate for the bridge deck, $S_{qr}(f)$, is given by

$$S_{qr}(f) = \frac{|X(f)|^2 |H_r(f)|^2}{\omega_r^4 M_r^2} S_{Fzr}(f) \cdots\cdots (12)$$

where $|H_r(f)|^2$ is the mechanical admittance in which the modal damping is replaced with the sum of the aerodynamic damping and structural damping, and $S_{Fzr}(f)$ is the power spectrum of the generalized buffeting lift force on the whole bridge deck obtained by Eq. (9) or (10).

The theoretical aerodynamic admittance between the w-component and the lift force for flat plates, which has been given by Sears, is used in the present study. It is approximately expressed[10] as

$$|X(f)|^2 = \frac{1}{1 + 2\pi^2 \frac{B_e f}{U}} \quad \cdots\cdots\cdots (13)$$

where B_e is the effective chord length, which is $B/\cos\beta$ for the cosine case and $l/\sin\beta$ for the sine case.

The square root of the integral of the response power spectra over the whole frequency range gives the r.m.s. generalized coordinate response, and the r.m.s. deflection of the bridge deck can be obtained by multiplying the mode shape at the point.

3 Numerical Examples

3.1 Tatara Bridge

3.1.1 Structural configuration and aerodynamic characteristics

Tatara bridge is a three-span cable-stayed bridge with a 890 m center span and a flat steel box girder shown in **Fig. 2**(a). The wind tunnel test was conducted for two construction stages (Stage 1, Stage 2) and the analytical procedure described above was applied for both stages. Stage 1 corresponds to the longest balanced cantilever spans where cantilever spans of 180 m long each are extended to both sides of the tower (Fig. 2(b)), and Stage 2 corresponds to the half bridge condition where the center span is just before closing and the side span is supported by piers (Fig. 2(c)). The model characteristics in the prototype scale for Stage 1 and Stage 2 are tabulated in **Table 1**. The generalized mass in the table is calculated for the whole structures.

The slope of the lift coefficient of the deck obtained through the section model test, $C_{L\alpha 2D}$, is 5.27 rad^{-1}, and the drag coefficient normalized with the deck width, C_{DC}, is 0.0725.

3.1.2 Results of the analysis

For the analysis, the characteristics of w fluctuating velocity component, I_w and L_x^w, are calculated by fitting the von Kármán spectrum to the power spectrum obtained from the experiment. The first and second mode are taken into consideration for the response calculation. The mode shape of Stage 1 is approximated as a straight line with a node at the tower for the first mode and as a polynomial of degree 4 for the second mode. For Stage 2, the side span is assumed to be motionless and the mode shape of the cantilever center span is approximated as the theoretical mode shape of a cantilever beam for the first mode and as that with some correction for the second mode.

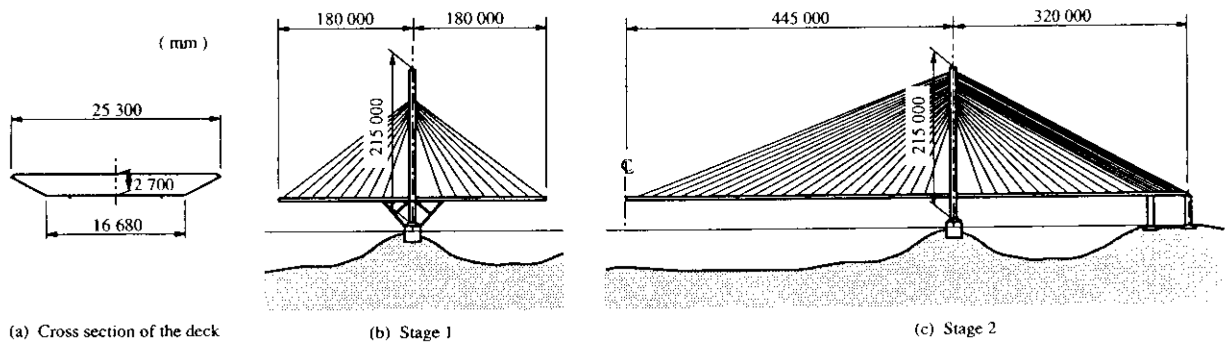

Fig. 2 Tatara bridge under construction

Table 1 Dynamic characteristics of the bridge model in prototype scale

	Stage 1			Stage 2		
	Natural frequency (Hz)	Generalized mass ($\times 10^3$kg)	Logarithmic damping	Natural frequency (Hz)	Generalized mass ($\times 10^3$kg)	Logarithmic damping
1st mode	0.172	2 207	0.018	0.242	653	0.018
2nd mode	0.662	272	0.020	0.313	560	0.019

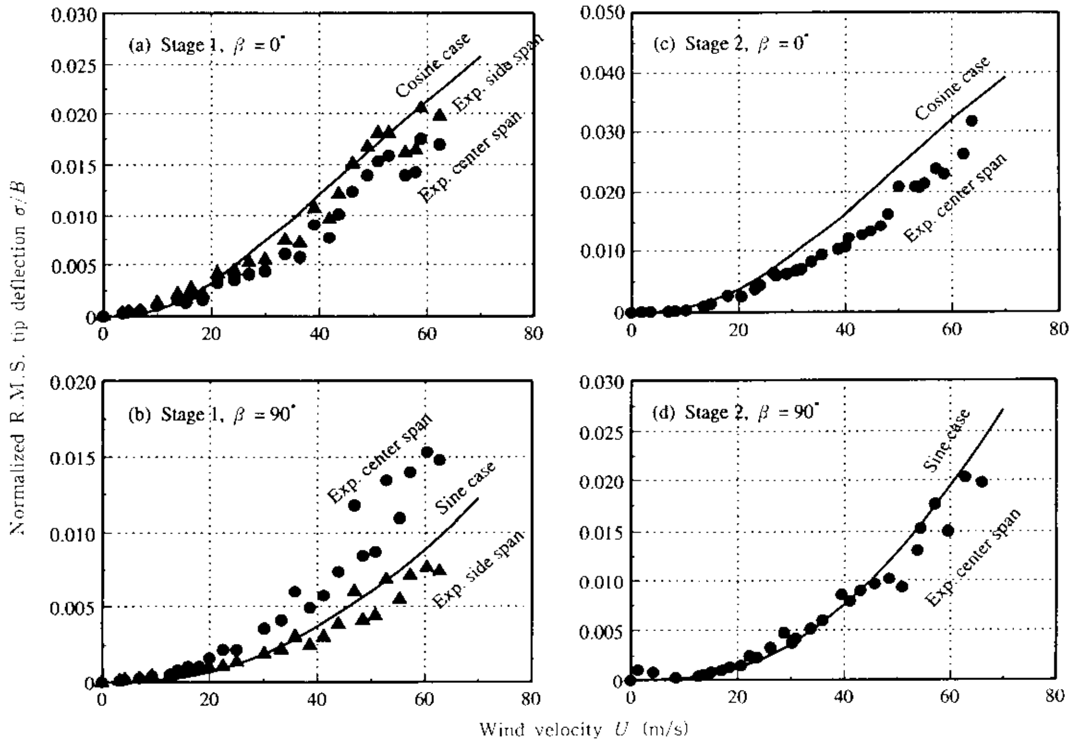

Fig. 3 Analytical and experimental results vs. wind velocity

The calculated r.m.s. vertical tip deflections against wind velocity for $\beta = 0°$ and 90° are compared with the experimental results in **Fig. 3**. The response is normalized with the deck width, and wind velocity is shown in prototype scale. The cosine case and sine case correspond to the case of $\beta = 0°$ and 90°, respectively. Only the result of the corresponding analysis case is shown in the figure. For Stage I, the measured response at the free end of the center span and side span are plotted with circle and triangle symbols, respectively.

The difference between the experimental tip deflection of the center span and the side span for Stage 1 with $\beta = 90°$ has been observed despite the fact that the first mode of vibration takes almost the same vertical deflection at both tips. It may be pointed out as the cause of the difference that the aerodynamic lift force acting upon the windward side (center span) is expected to be greater than on the leeward side, and the buffeting force acting on the tower may play some role in overall gust response, especially under yawed wind. However, the analysis gives the same response at both free ends for Stage 1 because symmetrical model characteristics and an anti-symmetrical mode shape with respect to the tower for the first mode, whose contribution to the response may be most significant, are assumed.

The analytical results of both cosine and sine cases generally reproduce the tendency of the increase of the response with wind speed, except for Fig. 3(b) where the experimental tip responses of the center span are much larger than the analytical values.

For the wind velocity of 40 m/s, the calculated responses in the yaw angle range between −90° and 90° are shown in **Fig. 4** with the measured ones for $\beta = 0°$, 10°, and 90°. The experimental results plotted in the figure are obtained by interpolation assuming the wind velocity and the response relationship as $\sigma/B = aU^2$.

The analytical predictions in Fig. 4 are about 20% greater than the experimental results for the $\beta = 0°$ and 10° cases, while they are closer for the $\beta = 90°$ case. Because only three wind yaw angle cases were tested, an extensive comparison regarding the effect of the wind yaw angle is difficult. Nevertheless, Fig. 4 indicates that its effect to the bridge response can be evaluated by this analysis.

3.2 Kao Ping Hsi Bridge

3.2.1 Structural configuration and aerodynamic characteristics

The Kao Ping Hsi Bridge is a single-tower cable-stayed bridge with a main steel span of 331 m and a concrete side span of 183 m, as shown in **Fig. 5**. The deck is 34.5 m wide and 3.2 m deep. Because of its asymmetric configuration with respect to the tower, which is 183 m high, the bridge has a very long cable-supported cantilever during its construction. The construction stage just before the closing of its main span is analyzed in the present study. The wind tunnel tests of this bridge have

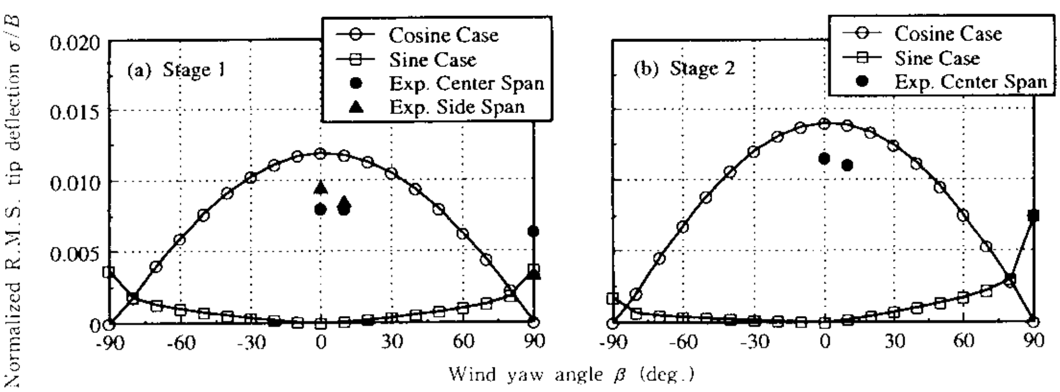

Fig. 4 Analytical and experimental results vs. wind yaw angle

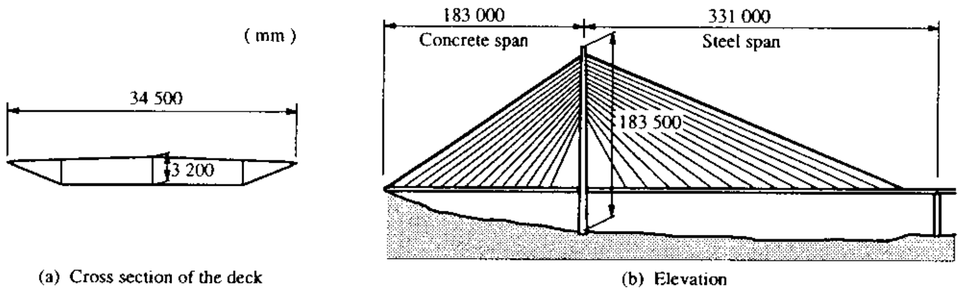

Fig. 5 Kao Ping Hsi Bridge

Table 2 Dynamic characteristics of the bridge model in prototype scale

	Natural frequency (Hz)	Generalized mass ($\times 10^3$kg)	Logarithmic damping
1st mode	0.190	1 651	0.020

been carried out at the Martin Jensen Wind Tunnel of the Danish Maritime Institute (DMI). The model characteristics for the tests are given in **Table 2**. In the calculation of the generalized mass in the table, only the contribution of the deck is taken into account and that of the tower and cables are neglected.

The slope of the lift coefficient of the deck obtained through the section model test, $C_{L\alpha 2D}$, is 6.0 rad^{-1}, and the drag coefficient normalized with the deck width, C_{DC}, is 0.0928.

3.2.2 Results of the analysis

The characteristics of w fluctuating velocity component, I_w and L_x^w, are obtained as in the previous example. However, the response is computed for the first mode only, and the mode shape is approximated as a polynomial of degree 5.

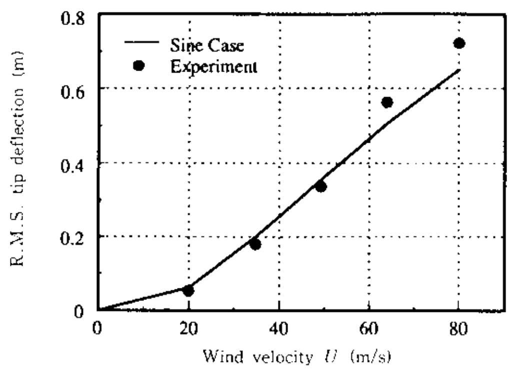

Fig. 6 Analytical and experimental results vs. wind velocity

The calculated r.m.s. tip deflections at some wind speed for $\beta = 0°$ are shown in **Fig. 6** with observed results. Both the deflection and the wind velocity are shown in prototype scale. Again, the analysis generally reproduces the tendency of the increase of the response with the wind velocity.

Figure 7 shows the comparison of the response with respect to the wind yaw angle in the analysis and experiment in the case of $U = 49.4$ m/s. The analytical results agree reasonably well with the test results regarding the change of the response with wind yaw angle.

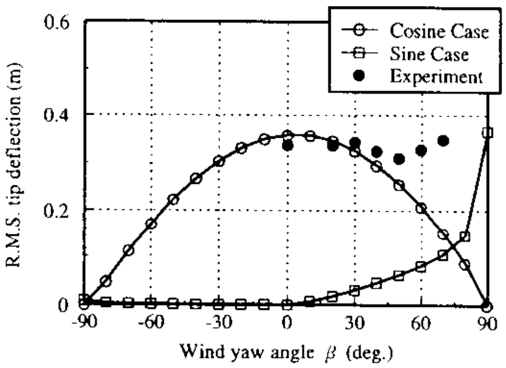

Fig. 7 Analytical and experimental results vs. wind yaw angle

cosine and sine cases in this study, and these cases correspond to only $\beta = 0°$ and $90°$ in the strict sense. Hence, the contribution from each case to the overall response in the intermediate range of wind yaw angles should be clarified.

5 Acknowledgments

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4 Conclusion

In the present study, the modified vertical gust response analysis proposed for a cantilever beam with a flat plate cross section in yawed wind was applied to cable-stayed bridges in their erection stages. By comparing the calculated responses with experimental observations, the validity of these assumptions and approximations was examined. The conclusions obtained from this study and the recommendations for further research are summarized as follows.

(1) Although some necessary coefficients for the analysis, such as C_{LaS} and aerodynamic damping, were approximately estimated from the aerodynamic characteristics of a flat plate with the same aspect ratio, the calculated responses agreed fairly well with the experimental ones. This confirms the validity of the approximations for cable-stayed bridges with a shallow deck.

(2) Since the results with these simple approximations were generally close to the experimental results, the method is considered to be applicable to response prediction in the early stage of the design.

(3) In order to improve the accuracy of the prediction, the adequacy of these approximations needs to be confirmed.

(4) The applicability of the Sears function as the aerodynamic admittance is also questionable, especially for the sine case, because the three-dimensional effect of the flow should be large. More work is needed on this point as well.

(5) The response was calculated separately for the

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