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Three-Dimensional Analysis of Flat Rolling of Rectangular Stock by a Numerical Method Based on Upper Bound Theory*

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1 Introduction

Recent years have seen such remarkable progress in rolling theory that analyses based mainly on the slab method have been considered almost complete in the field of plate rolling. However, due to restrictions inherent in the slab method, these analyses have mainly been applied to plane-strain rolling. It cannot be said that problems related to three-dimensional deformation such as bar and shape rolling are thoroughly analyzed. Large-scale simulation such as that by the finite element method is effective in solving such problems of large deformation. In treating three-dimensional deformation, however, the finite element method sometimes poses problems of computer memory size and computation time. Small-scale computations are sometimes effective.

In addition to the finite element method, the upper bound technique is also available. If the shape of deformation can be expressed by a simple velocity field, a velocity field close to the true value can be obtained by adjusting parameters in a function expressing the velocity field. Reported analyses by the upper bound technique include a method in which a velocity field approximated by a cubic function is optimized using the Ritz method[1], and methods in which nonlinear optimization is conducted to obtain an optimum velocity field[2,3].

The authors attempted to analyze the flat rolling of a rectangular bar by the latter method of nonlinear optimization[4]. The rolling of rectangular bars with flat rolls to produce wire rod and bars is a new rolling method in the rougher mill that has been put into practical use as a type of grooveless rolling[5]; because of the relatively simple shape of deformation, it is a working method suitable for analysis by the upper bound technique.

This report gives a comparison between calculation results and results of lead-model experiments, as well as results of an examination of deformation energy efficiency based on the calculation results.

2 Analysis Method

Principal assumptions made for the analysis are shown below:

(1) The material is a rigid perfectly plastic solid.
(2) The material is noncompressible.
(3) The cross section of the stock remains plane during rolling.
(4) Rolls used in rolling are rigid.

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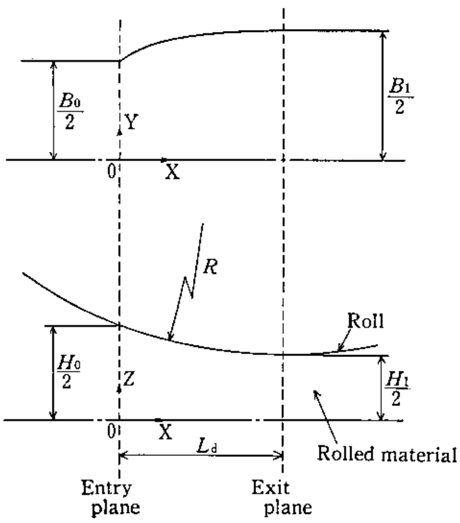

Fig. 1 Coordinates of flat rolling analysis

(5) The tangential stress τ on roll surfaces is given by $\tau = mk$ (m is the friction constant and k is the shear yield stress).

(6) Bends and cambers do not occur in the rolled steel.

If coordinates and symbols as shown in **Fig. 1** are used and a certain increment of deformation occurs in supposed steady deformation, then the following formula holds in accordance with the upper bound theorem[6]:

$$2T\omega \leq \int_V \sqrt{3}\, k \dot{\varepsilon}_{eq}^{k} dV + \int_{\Gamma k} k \Delta v_T^k d\Gamma + \int_S \tau \Delta v_T^k dS \cdots\cdots(1)$$

where T: Torque for one roll (kgf·mm)

ω: Angular velocity around a roll (rad/s)

k: Shear yield stress (kgf/mm^2)

$\dot{\varepsilon}_{eq}$: Equivalent plastic strain rate (s^{-1})

Γ_k: Plane of discontinuity of velocity

S: Surface in contact with a roll

Δv_T: Amount of discontinuity of velocity–relative slip velocity (mm/s)

From left to right the terms on the right side of Eq. (1) represent energy dissipation rates by plastic deformation, discontinuity of velocity, and friction. The superscript k represents a kinematically admissible state.

It is supposed that the external shape, i.e., shape of free surface w during steady rooling, is expressed by a function of longitudinal position x only, and that this shape is approximated by a cubic function[1]. Furthermore, the cross section before rolling keeps a plane, as mentioned in assumption (3), and it is assumed that the shape of the free surface is uniform in the thickness direction. As shown in Fig. 1, the entry plane is denoted by $x = 0$, the exit plane by $x = L_d$, and the stock height and width before rolling by H_0 and B_0, respectively.

Since $w = B_0$ at $x = 0$, $w = B_1$ at $x = L_d$, and $dw/dx = 0$ at $x = L_d$, the external shape is given by the following equation:

$$w(x) = B_0 + C_0 x + (3 \cdot w_0 \cdot B_0 - 2C_0 L_d)\left(\frac{x}{L_d}\right)^2 + (C_0 L_d - 2w_0 B_0)\left(\frac{x}{L_d}\right)^3 \cdots\cdots(2)$$

C_0 represents the ratio of change of the flow line of the external shape in the entry plane; w_0 represents the spread ratio in the exit plane.

The stock height at a longitudinal position x is given by Eq. (3).

$$H(x) = H_1 + 2R - 2\sqrt{R^2 - (L_d - x)^2} \cdots\cdots(3)$$

where R is the roll radius and H_1 is the stock height on the exit side. Since position x_n of the neutral point is also unknown, it is taken as the adjusting parameter.

When these variables are given, the stock velocity U_n at the neutral point in the x-direction is expressed by the following equation using the roll radius R and angular velocity around a roll ω:

$$U_n = \frac{R\omega}{\sqrt{1 + H'(x_n)^2}} \cdots\cdots(4)$$

Since the mass flow is constant, the longitudinal stock velocity v_x at a point in the longitudinal direction (x-direction) is given using U_n as follows:

$$v_x = \frac{U_n \times H(x_n) \times w(x_n)}{H(x) \times w(x)} \cdots\cdots(5)$$

Since the transverse velocity v_y is zero at the middle of the stock width and changes uniformly toward the transverse edges in accordance with assumptions (3) and (6), v_y is given by the following equation:

$$v_y = v_x \times w'(x) \times \frac{y}{w(x)} \cdots\cdots(6)$$

Since the velocity v_z in the thickness direction is zero at the middle of the stock thickness and changes uniformly toward the surfaces in contact with rolls in accordance with the assumptions (3) and (6), v_z is given by the following equation:

$$v_z = v_x \times H'(x) \times \frac{z}{H(x)} \cdots\cdots(7)$$

In other words, velocities in the x-, y- and z-directions in each part of the stock being deformed can be found if three variables, i.e., the parameters expressing external shape C_0 and w_0 and the parameter expressing the position of neutral point x_n, are given. The plastic strain rate and equivalent plastic strain rate can be found from these velocity fields by Eq. (8)[7], and the energy dissipation rate can be calculated from these strain rates. In this study, these calculations were made by the numerical integration of the analysis region.

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

$$\dot{\varepsilon}_{eq} = \sqrt{\frac{2}{3}\sum_{ij=1}^{3} \dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$$

The velocity field nearest to the true value can be obtained if a stationary condition for the total energy dissipation rate Φ and the steady condition, in which the velocity field after the satisfaction of this stationary condition coincides with the initially assumed velocity field, are both satisfied by appropriately selecting these three variables to express the external shape.

In this report, the simplex method[8], which is one direct calculation method for nonlinear optimization, was used for the optimization of results of calculation by the upper bound technique. In the simplex method, function values at apexes of simplexes are compared and the search direction is determined by repeating the reflection, expansion, and contraction of the simplexes, as shown in **Fig. 2**. When the number of degrees of freedom is small, the convergence characteristic is usually good, making this an effective calculation method. The flow of calculation is schematically shown in **Fig. 3**.

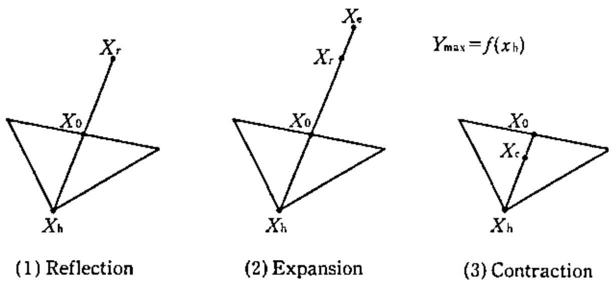

Fig. 2 Schematic representation of direct searching by simplex method

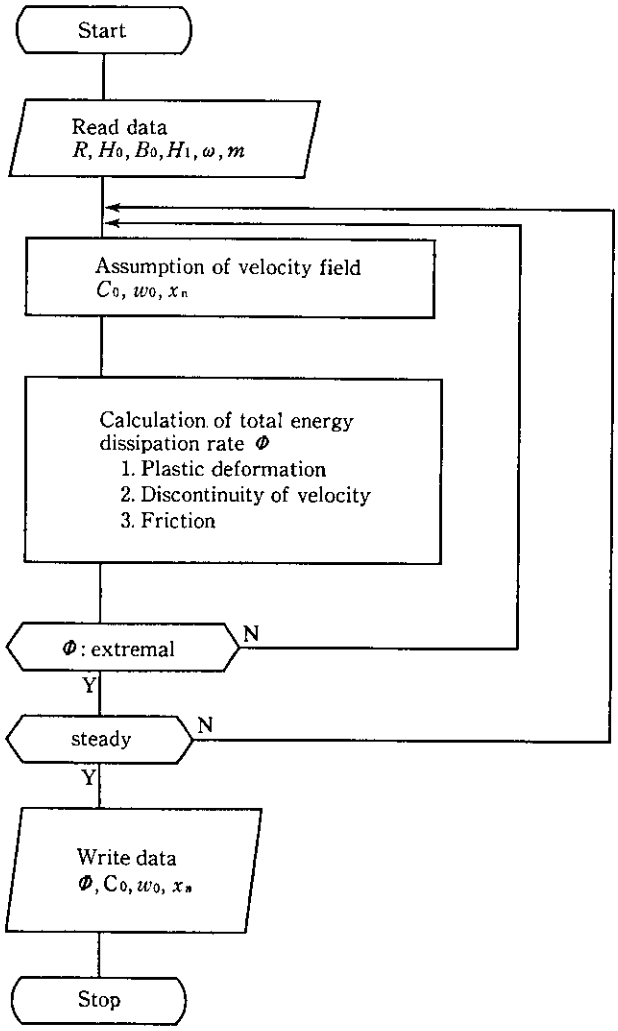

Fig. 3 Flow chart of calculation

3 Experimental Method

A comparison was made between results of an analysis by the upper bound technique and those of lead-model experiments[9] using a stock of almost the same width and thickness as that of a rectangular bar. Flat rolling of a rectangular bar corresponds to grooveless rolling and has recently attracted attention as a new rough rolling method for wire rods and bars. Conditions used in calculation and the experiment are given in **Table 1**.

Table 1 Conditions of calculation and experiment

Roll diameter	200 mmϕ
Material	99.99% Pb
Stock height H_0	40, 30, 23, 18 mm
Width/height B_0/H_0	0.5, 1.0, 2.0
Reduction	10, 20, 30, 40%

The selection of these conditions was based on the characteristics of the rougher and intermediate mill in the bar mill at Kawasaki Steel's Mizushima Works. A 1/3 scale model was used in the experiment. Although the calculation conditions selected were equivalent to the experiment conditions, the range of stock width and friction conditions in the calculation conditions was wider than the experimental range.

The material for the model used in the experiment was lead of 99.99% purity, which recrystallizes at room temperature and is considered to be a good stock for simulating the hot rolling of steel[10]. The rolling experiment using the model was conducted at room temperature. After each rolling pass, the roll surface was ground with emery paper (# 300) and then degreased with benzine to prevent a decrease in the coefficient of friction due to the adherence of lead. The reduction ratio was varied between 10% and 40%. At high reduction ratios, in some cases the leading end of the stock did not enter rolls. On such occasions, the leading end of the stock was pre-rolled to reduce its thickness and was then fed into the rolls. In the present range of experimental conditions, slip did not occur during steady rolling.

When pure lead is rolled, irregularities on the free surface after rolling become marked, making accurate size measurement difficult. For this reason, before rolling stocks were scribed with gauge lines and the average width after rolling was found by determining elongation from changes in distance before and after rolling.

4 Comparison between Experiment and Calculation Results; Discussion

The relationship between the spread ratio and reduction determined from the average width is shown in **Fig. 4**. In the rolling of a stock of almost the same width and thickness as in this experiment, the width after rolling increased exponentially with increasing reduction, as already reported[11]; the smaller the stock height H relative to the roll diameter D, i.e., the higher the ratio D/H, the more pronounced this tendency becomes.

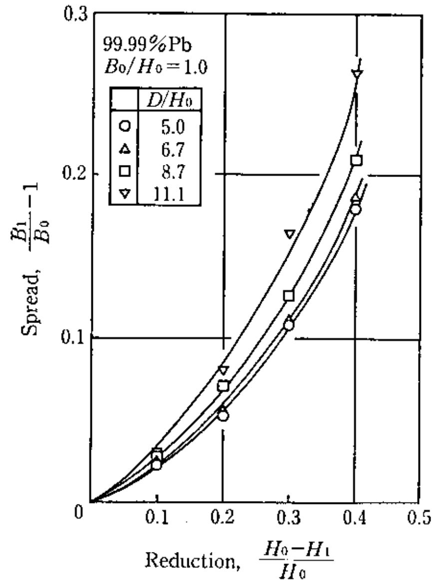

Fig. 4 Relationship between reduction and spread (experimental)

Various empirical formulas for expressing spread during the rolling of wire rods and bars have been proposed[12,13,14]. Among others, Shinokura's equation[15], in spite of its simplicity, is considered applicable to various groove-type rolling methods.[16] Shinokura's equation is used to predict stock width after rolling on the basis of the geometrical shape before rolling, as illustrated in **Fig. 5**.

$$\frac{B_1}{B_0} - 1 = \alpha \times \frac{L_d}{H_0 + 2B_0} \times \frac{A_H}{A_0} \quad \cdots\cdots\cdots\cdots (9)$$

where A_H: Cross-sectional area outside the groove (mm^2)

A_0: Cross-sectional area before rolling (mm^2)

α: Experiment coefficient

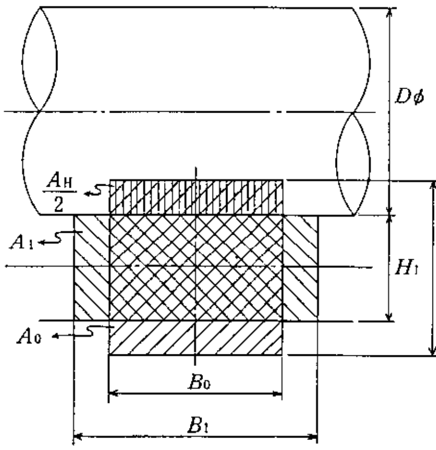

Fig. 5 Schematic view of flat rolling of rectangular stock

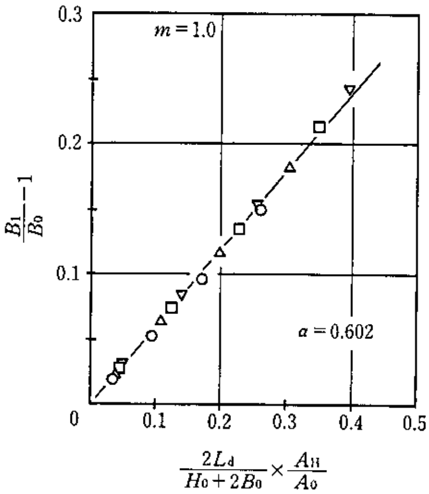

Fig. 7 Relationship between Shinokura's spread equation and width spread (calculated $m = 1$)

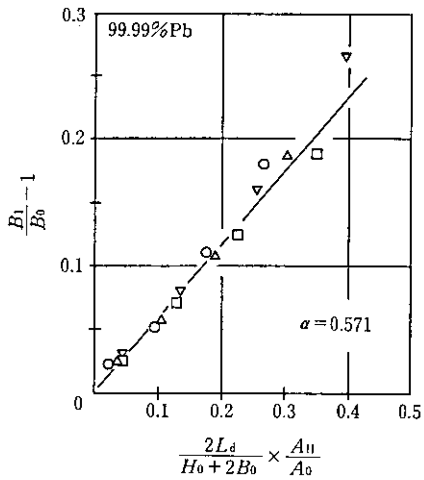

Fig. 6 Relationship between Shinokura's spread equation and width spread (experimental)

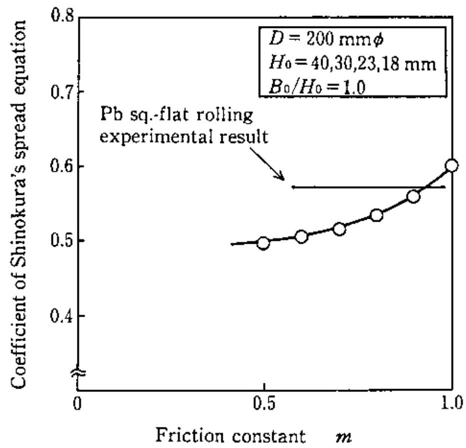

Fig. 8 Relationship between friction constant and coefficient of Shinokura's spread equation

The spread ratio of the square bar being flat-rolled in this experiment can be reexpressed in Shinokura's equation as shown in **Fig. 6**. It is apparent from this figure that results of an experiment conducted with various stock heights and reductions are approximated by a straight line. The slope of the correlation line intersecting the origin is found to be 0.571 from experimental values by the least square method. This value corresponds to the coefficient α in Shinokura's spread equation.

Results of calculation by the upper bound technique at the friction constant m of 1.0 are similarly graphed in **Fig. 7**. These calculated results can also be approximated by a straight line intersecting the origin when expressed by Shinokura's equation. The slope of this line is 0.602, or almost equal to the above-mentioned value obtained in the lead-model experiments. The friction constant m of 1.0 means that $\tau = k$; that is, the stock is subjected to a shear yield stress on the roll surface (sticking). Since the slope value α in Shinokura's equation is almost equal to the α value in calculations of spread for $m = 1.0$, it was surmised that a state very close to sticking obtained in this experiment.

To further investigate this matter, the slope α in Shinokura's equation was determined by varying the friction constant m from 0.5 to 1.0 in 0.1 steps. Results of this calculation are shown in **Fig. 8**. It is apparent that the slope in Shinokura's equation decreases as the friction constant m decreases from 1.0, and that this tendency to decrease becomes less marked as the constant m approaches 0.5. In other words, spread decreases with decreasing friction constant; an equivalent finding was reported elsewhere[17]. The α value of 0.571 obtained from the lead-model experiment corresponds to a fric-

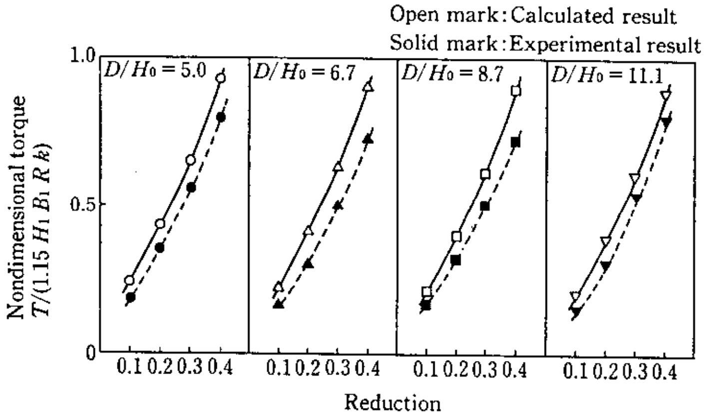

Fig. 9 Accuracy of calculation results of rolling torque

tion constant m of 0.93 when it is estimated from Fig. 8; thus it may be said that the friction conditions in the experiment were quite close to sticking.

A comparison between calculated results for rolling torque and measured values is given in **Fig. 9**. In this figure, the ordinate represents non-dimensional torque values obtained by dividing torque by the cross-sectional area after rolling $H_1 \cdot B_1$, the roll radius R, and the shear yield stress k. Dashes denote calculated results and the solid lines denote measured results. The plotted data in the four sections of this figure represent stock heights of 40, 30, 23 and 18 mm, respectively, from left to right. In each case, calculated values were in virtual agreement with measured values, although the former tend to be higher than the latter. The reason the calculated values were higher than the measured values is that in the upper bound technique used, calculated values cannot be lower than the true value.

When these results calculated by the upper bound technique were compared with results of the lead-model experiment without lubrication, good agreement was observed in both the spread ratio and torque at m of 1.0, confirming the validity of the calculations.

5 Examination of Deformation Energy Efficiency

Since the energy dissipation rate during rolling can be determined by the upper bound technique, it is possible to examine effects of various rolling conditions on deformation energy efficiency.

One of the principal purposes of rolling is to elongate materials. However, dissipation of deformation energy not contributing to this elongating is required to some degree due to the additional shearing deformation during rolling. For comparison based on a case where the same elongation is given, deformation energy efficiency η is given by the following equation[18], when rolling energy per unit volume is denoted by a_w and the energy per unit volume required to obtain the same elongation given by uniaxial tension is denoted by $a_w{}^*$:

$$\eta = \frac{a_w{}^*}{a_w} \quad \cdots\cdots(10)$$

$$a_w{}^* = K_{fm} \cdot \ln \lambda \quad \cdots\cdots(11)$$

$$a_w = \frac{2T}{A_1 \cdot R \cdot (1 + \phi)} \quad \cdots\cdots(12)$$

where η: Deformation energy efficiency
$a_w{}^*$: Ideal deformation energy (kgf/mm^2)
a_w: Energy per unit volume (kgf/mm^2)
K_{fm}: Mean deformation resistance (kgf/mm^2)
λ: Elongation ($= l_1/l_2$)
A_1: Cross-sectional area after rolling
ϕ: Forward slip

Since a_w includes the additional shearing deformation energy, friction energy, etc. present during rolling, this value is higher than $a_w{}^*$, and η usually shows values lower than 1. If the value of η is high, the energy necessary for giving the same elongation to a stock being rolled is low. Therefore, high η values are desirable for reducing rolling power requirements.

Effects of stock height and width on deformation energy efficiency are shown in **Fig. 10** and **11** respectively. In these figures, reduction is plotted as the ordinate, and deformation energy efficiency as the abscissa. It is apparent from these figures that the larger the stock height relative to roll diameter, the higher the deformation energy efficiency. This tendency is great at high reductions. Furthermore, for stocks of the same height, the larger the stock width, the higher the deformation energy efficiency.

These phenomena can be explained mainly by the spread tendency of the rolled stock. The greater the stock height relative to roll diameter and the wider the stock relative to stock height, the smaller will be the

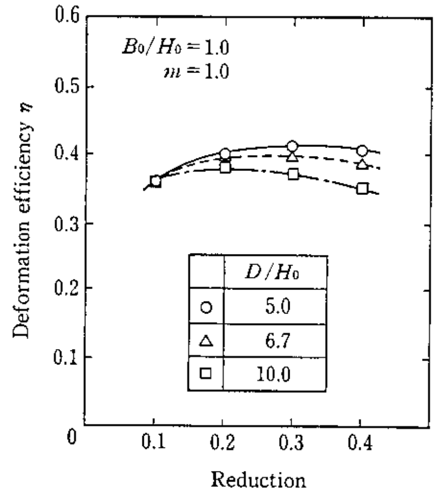

Fig. 10 Effect of stock height on deformation energy efficiency

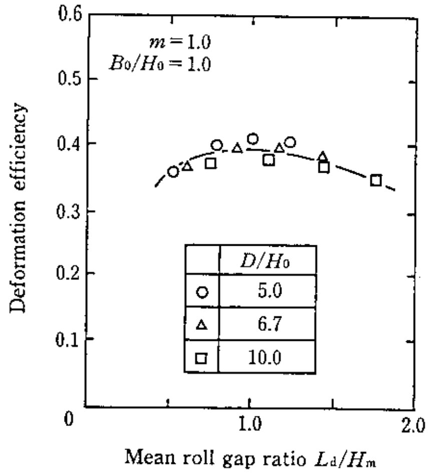

Fig. 12 Relation between mean roll gap ratio L_d/H_m and deformation energy efficiency

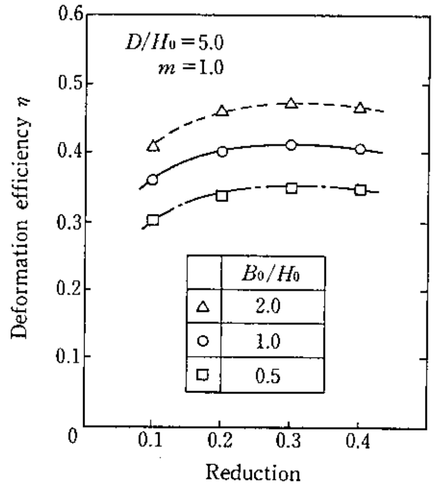

Fig. 11 Effect of stock width on deformation energy efficiency

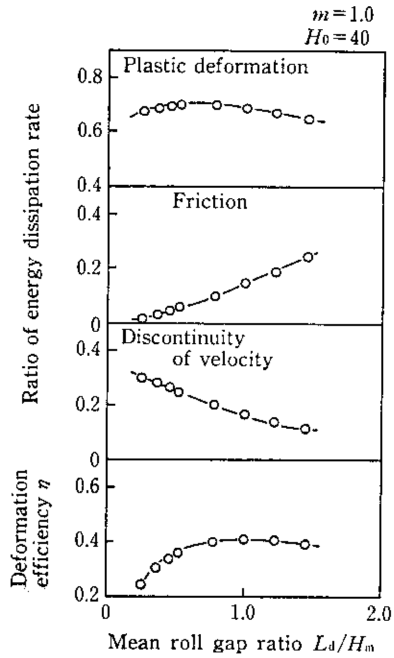

Fig. 13 Effect of each energy dissipation rate on deformation energy efficiency

spread of a stock being rolled. Deformation energy efficiency increases because the elongation of the stock is large at the same reduction ratio, for the reason mentioned above.

This deformation nergy efficiency η can be expressed by the mean roll gap ratio L_d/H_m using the mean stock thickness H_m given by Eq. (13), as shown in **Fig. 12**.

$$H_m = \frac{H_0 + H_1}{2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (13)$$

If the ratio of stock width to stock height is constant, η can be expressed with good accuracy by a straight line with respect to L_d/H_m, and shows maximum values ranging from 0.35 to 0.4 at L_d/H_m of about 1.0. The reason η shows peaks with respect to L_d/H_m seems to be that the ratios of energy dissipation rates by plastic deformation, friction, discontinuity of velocity, etc. to the total energy dissipation rate change with rolling conditions. These ratios were calculated for D/H_0 of 5; results of the calculation are shown in **Fig. 13**. In this figure, the ratios of energy dissipation rates by plastic deformation, friction, and discontinuity of velocity to total energy dissipation rate and deformation efficiency are plotted on the ordinate downward from the top. It is evident from this figure that the ratio of energy dissipation by friction increases with increasing L_d/H_m and that the ratio of energy dissipation by discontinuity of velocity increases with decreasing L_d/L_m. In other words, under rolling conditions with L_d/H_m not close to 1.0, the degree of increase in either the ratio of energy dissipation by friction to the total energy dissipation rate, or that by the discontinuity of velocity, will be larger than the degree of decrease in the other. Therefore, the ratio of energy dissipation by plastic deformation will show a relative decrease. This seems to explain why η shows a peak at L_d/H_m of about 1.0.

The foregoing suggests that to optimize the deformation energy efficiency in grooveless rolling, it is necessary that rolling conditions such as roll diameter and reduction be such that the value of L_d/H_m is close to 1.0. In the rolling of wire rods and bars, both the stock thickness and width are reduced. Therefore, if the aspect ratio after a pass is excessive, the ratio H/B at the next pass increases and the extent of spread becomes large, resulting in a decrease in deformation energy. Accordingly, it is important to optimize the total deformation energy efficiency in consideration of these points.

6 Conclusions

Results of a three-dimensional analysis of rolling of a rectangular bar by the upper bound technique were compared with those of lead-model experiments and the deformation energy efficiency was examined using the calculation results. The following results were obtained.:

(1) The analysis results were compared with the experiment results obtained from the rolling of lead without lubrication. Both showed almost the same spread at the friction constant m of 1.

(2) Results of calculation of torque were in near agreement with the experiment results of lead rolling, though the former showed slightly higher values.

(3) Deformation energy efficiency shows maximum values at specific reductions. The larger the stock height relative to the roll diameter and the larger the stock width relative to the stock height, the higher the deformation energy efficiency will be.

(4) The deformation energy efficiency η shows a peak at L_d/H_m of 1.0. Under rolling conditions with L_d/H_m not close to this value, η decreases because the energy dissipation rate by friction or discontinuity of velocity increases.

These findings were obtained in spite of the assumption of a simple velocity field where the cross section before rolling keeps a plane. When more complex deformation phenomena are treated, the upper bound technique poses the problems of uniqueness and convergence of solutions in optimization, in addition the difficulties in describing the velocity field. For this reason, finite element method are being used[19,20,21], and research in this direction will be active in the future.

References

1) S. I. Oh and S. Kobayashi: *Int. J. Mech. Sci.*, **17**(1975)4, 293
2) T. Murota, K. Kato, K. Jinma, and K. Okaniwa: Proc. 26th. Japan. Joint. Conf. Tech. Plasticity, (1975), 29
3) Y. Saito, T. Morishita, and K. Kato: Proc. 1980. Japan. Spring. Conf. Tech. Plasticity, (1980), 223
4) H. Hayashi, K. Kataoka, H. Yoshida, and K. Isobe: *Tetsu-to-Hagané*, **68**(1982)12, S500
5) T. Yanazawa, T. Tanaka, A. Noda, R. Takeda, S. Morita, and H. Hayashi: *Kawasaki Steel Giho*, **14**(1982)3, 85
6) R. Hill: "The Mathematical Theory of Plasticity", (1950), [Clarendon Press]
7) K. Kato, T. Murota, and T. Kumagai: *J. Jpn. Soc. Tech. Plasticity*, **21**(1980)231, 359
8) L. C. W. Dixon: "Nonlinear Optimization", (1972), [English Universities Press]
9) H. Hayashi, K. Kataoka, A. Noda, and M. Inoue: *Tetsu-to-Hagané*, **68**(1982)5, S384
10) S. Shida, H. Awazuhara, K. Yasuda, and S. Tsumura: *J. Jpn. Soc. Tech. Plasticity*, **19**(1978)212, 796
11) S. Yanagimoto: *J. Jpn. Soc. Tech. Plasticity*, **2**(1961)6, 37
12) M. L. Zaitsev: *Stal in English*, Dec. (1968), 1037
13) M. Vater and A. Schutza: *Arch. Eisenhüttenwesen*, **42**(1971)7, 459
14) V. I. Nikitin and V. K. Smirnov: *Steel in USSR*, **4**(1974)4, 310
15) T. Shinokura and A. Katsuyama: Proc 29th Japan. Joint. Conf. Tech. Plasticity, (1978), 109
16) Y. Saito, S. Nakano, and Y. Takahashi: Proc. 1979. Japan. Spring. Conf. Tech. Plasticity, (1979), 505
17) K. Kato, Y. Saito, Y. Takahashi, and S. Yoshiki: Proc. 1975. Japan. Spring. Conf. Tech. Plasticity, (1975), 25
18) Y. Saito, M. Moriga, Y. Takahashi, and K. Kato: Proc. 1980. Japan. Spring. Conf. Tech. Plasticity. (1980), 5
19) K. Kato, T. Murota, and K. Komori: *Trans JSME.(A)*, **51**(1985)469, 2172
20) Y. Saito, O. Shinshi, K. Kato: Proc. 33rd. Japan. Joint. Conf. Tech. Plasticity, (1982), 41
21) Y. Saito and Y. Kusumoto: *Nakayama Steel Technical Report*, **23**(1984), 25