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Yuji Tanaka, Kazuo Omori, Takanori Miyake, Katsumi Nishizaki, Masatoshi Inoue, Sakae Tezuka

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The body can be viewed from the next page.

Camber Control Techniques in Plate Rolling*

Yuji Tanaka
Staff Assistant Manager, Electrical & Instrumentation Technology Sec., Mizushima Works

Kazuo Omori
Staff Assistant Manager, Development & Design Sec., Equipment Dept., Mizushima Works

Takanori Miyake
Staff Assistant Manager, Plate Control Sec., Mizushima Works

Katsumi Nishizaki
New Materials Research Center, High-Technology Research Laboratories

Masatoshi Inoue
Staff Manager, Plate Rolling, Casting & Forging Technology Sec., Mizushima Works

Sakae Tezuka
Staff Manager, Silicon Dept. New Business Div.

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New, unique measuring and control techniques of plate camber have been developed and utilized in the plate mill of Mizushima Works. Features of the techniques are as follows:

(1) Plate camber can be measured regardless of sideslipping or turning of a plate during rolling.

(2) Any nonuniform plate camber profile can be accurately expressed in the form of the n-th order polynomial.

(3) The measured camber is used to calculate the aimed wedge, to which the actual wedge is fed back by a feedback control system, thereby reducing the plate camber in the next pass.

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1 Introduction

Control of plate camber (edgewise curvature) in rolling is one of the largest technical problems yet to be solved in the field of rolling. Camber results in lowering of yield and causes poor grading and lower efficiency in the shearing process.

In the past, a method of preventing happenings of camber during rolling was such that a rolling operator adjusted the difference of roll gaps between the right side and the left, the instant that he noticed the presence of camber on his visual inspection. Relying solely on the sixth sense and experience of the operator, this method fell short of correcting cambers completely.

As a result, many studies have been made in recent years to pursue the camber generation mechanism in the rolling and further in controlling the camber.[1-7] At present the method of preventing the camber is classified into two kinds.

One is meandering control.[4,5] This is a method of suppressing camber by preventing the occurrence of sideslipping (slipping of the plate-roll contacting position in rolling) as the cause of camber. This method is useful for the strip mill. The defect in this method lies in the impossibility of correcting the camber which has already been generated.

The other is a method in which camber is measured during rolling so as to correct it in the subsequent rolling pass.[6-8] Since this method effects control after measurement, it is suitable to the reversing mill like a plate mill.

At the plate mill of Mizushima Works, studies have been underway on the camber control using the latter method. As a result, Mizushima Works has developed methods of its own for camber measuring and controlling. This camber control has already been incorporated into the plate rolling process, and is contributing to yield improvement, reduction in poor grading, and efficiency increase in the shearing process.

In this paper, a report is made on the camber measuring and controlling methods in the plate mill as well as the results of their application to a commercial plate rolling.

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2 Camber Measurement

In camber measurement, the following problems were posed:

(1) Measuring must be made at a point nearest to the mill in order not to lower rolling efficiency.
(2) Measuring must be free from the influence of sideslipping of the plate during rolling, or anything similar.
(3) Measuring must express faithfully the camber curvature distribution in the longitudinal direction, because this camber curvature is not always uniform in the whole plate length.
(4) Measuring must express the measured values in a simple function form, so that the measurement can be immediately used for the subsequent control.

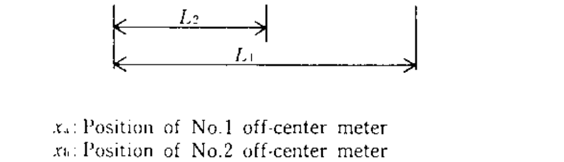
At Mizushima Works, three devices to measure the central position in the width direction of the plate (hereinafter called the "off-center meters") are installed in the length direction of the rolling line. And, by expressing the camber profile of the plate by the nth order polynomial, the above-mentioned problems have been solved. Details of this procedure are described below.

2.1 Principle of Measurement

Principle of measurement is explained on the basis of **Fig. 1**. Assume that, in the figure, the x-axis and y-axis correspond to the length and width directions of the plate, respectively, and that off-center meters are installed each at locations x_a, x_b, and x_c. Distance between x_a and x_c and between x_a and x_b are denoted by L_1 and L_2, respectively.

Now it is supposed that the camber profile of a plate can be expressed by the following nth order polynomial:

$$y = F(x) = a_0 + a_1 x + \cdots + a_n x^n \quad \cdots\cdots (1)$$



x_a: Position of No.1 off-center meter
x_b: Position of No.2 off-center meter
x_c: Position of No.3 off-center meter

Fig. 1 Principle of measurement

During rolling, this plate progresses in the positive direction of the x-axis, while sideslipping and rotating occationally. To simplify the explanation, it is assumed here that there is no such sideslipping and rotation.

Assume that the position of a plate measured by three off-center meters is expressed by three points A, B and C, and that a straight line l passing A and B crosses another straight line $x = x_c$ at D. At this time, the distance M between C and D is expressed as follows:

$$M = F(x_c) - H(x_c) \quad \cdots\cdots (2)$$

where $y = H(x)$ is an equation of straight line l and is expressed as follows:

$$H(x) = \frac{F(x_b) - F(x_a)}{L_2} x + \frac{F(x_a)x_b - F(x_b)x_a}{L_2} \quad \cdots\cdots (3)$$

As the plate moves in the positive direction of the x-axis, the value of M changes. If the value of M—when the plate moves by s from the present position—is denoted by $G(s)$, curved line $m = G(s)$ will express changes in M brought by the motion of the plate. When Eqs. (1) and (3) are substituted in Eq. (2), and further, x_a, x_b and x_c are altered into $x_a - s$, $x_b - s$ and $x_c - s$, then $G(s)$ is obtained. And this will become the following $(n-2)$th order polynomial.

$$G(s) = b_0 + b_1 s + \cdots + b_{n-2} s^{n-2} \quad \cdots\cdots (4)$$

where coefficients $b_0, b_1, \ldots, b_{n-2}$ are expressed as the linear combination of coefficients $a_2, a_3, \ldots, a_n$. Namely,

$$b_i = \sum_{j=2}^{n} C_{ij}(x_a, L_1, L_2) \cdot a_j \quad \cdots\cdots (5)$$

$$(i = 0, 1, \ldots, n-2)$$

In the above equation, C_{ij} is a function of x_a, L_1 and L_2.

From the above, it is possible to obtain the polynomial $y = F(x)$, which expresses the camber profile, by the following procedure:

(1) To seek after M which changes as the plate moves. Then, changes in M are approximated by the $(n-2)$th order polynomial as shown in Eq. (4), and $b_0, b_1, \ldots, b_{n-2}$ are obtained.
(2) To solve simultaneous equations (5) in order to obtain $a_2, a_3, \ldots, a_n$.
(3) If the shape of the plate is expressed by Eq. (1), the following two equations become valid at two arbitrary measuring points x_1 and x_2:

$$F(x_1) = a_0 + a_1 x_1 + \sum_{j=2}^{n} a_j x_1^j$$
$$F(x_2) = a_0 + a_1 x_2 + \sum_{j=2}^{n} a_j x_2^j \quad \cdots\cdots\cdots(6)$$

Then two measured off-center values $F(x_1)$ and $F(x_2)$ and $a_2, a_3, \ldots, a_n$, which have been already derived, are substituted in Eq. (6). And the simultaneous equations are solved to obtain a_0, and a_1.

Now, in the explanation made heretofore, it was assumed that there was no sideslipping and rotation of the plate, but in the actual rolling operation, such movements occur at times. According to the present method, however, their adverse effects can be disregarded. The reasons for this are explained below.

First, no adverse effect is exercised by the sideslipping of the plate, because the value of M will not change, even if $y = F(x)$ makes parallel movements in the y-axis direction.

Next, the effect of rotation of the steel plate is minimal, so it can be disregarded. Rotation of the plate on the rolling table is for 0.3° at the most, and rotation to this degree will only bring about changes in the M value by mere 1 mm maximum.

Consequently, there is no possibility that the measuring error of M will exercise an adverse effect on the estimation of the coefficient of $y = F(x)$. In the simulation separately conducted, which is omitted in this report, it has been confirmed that the effect of rotation is negligible.

2.2 Configuration of the Equipment

The arrangement of instruments for the camber meter (camber measuring equipment) is shown in **Fig. 2**. The instruments consist of three off-center meters. Their specifications are shown in **Table 1**. In **Fig. 3**, the system configuration of the camber meter is given.

The central position in the width direction of the plate is measured by three off-center meters, and on the basis of the measurements, distance M is calculated by an upper-rank microcomputer. This distance value is transmitted to the process computer, where first it is approxi-

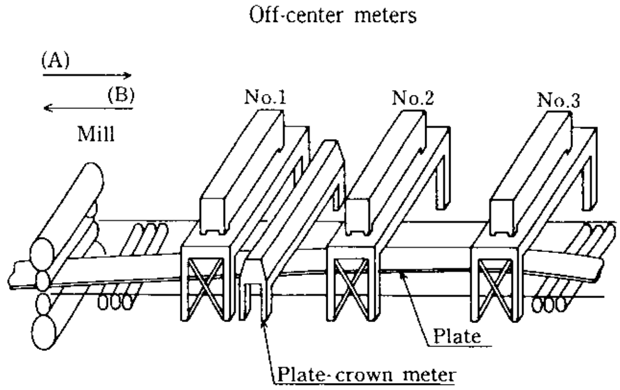


Fig. 2 Arrangement of instruments for camber measurement and control

Table 1 Specifications of off-center meter

Function	Performance
Measuring range of plate width	1 000 — 5 500 mm
Measuring range of off-center position	±150 mm
Accuracy	± 2 mm

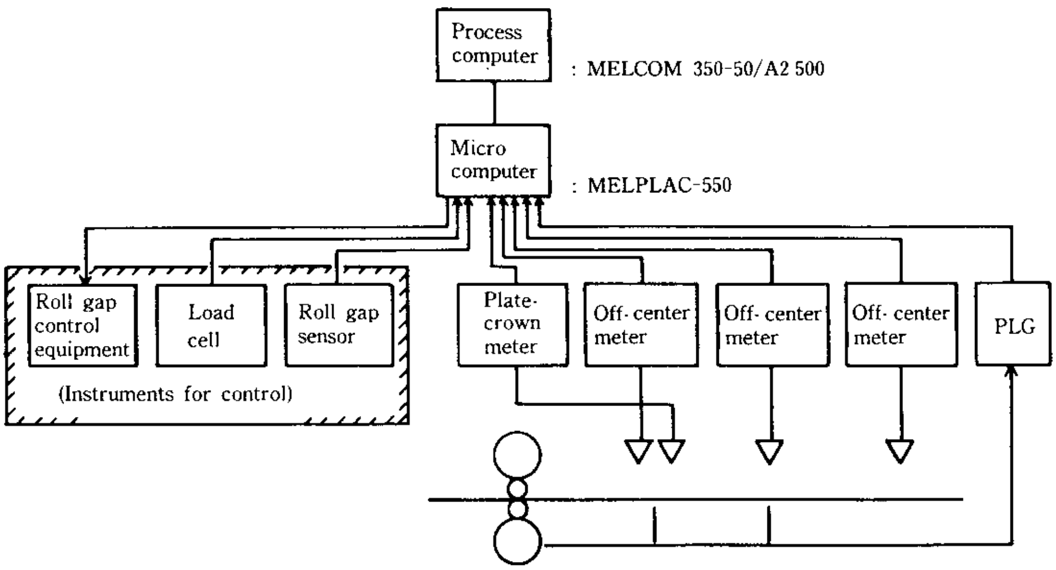


Fig. 3 System configuration for camber measurement and control

mated to the $(n-2)$th order polynominal, and then coefficients $a_0, a_1, \ldots, a_n$ of the nth order polynomial $y = F(x)$, which expresses plate camber, are calculated. The camber profile thus determined is fully used in camber control and displayed in patterns on the CRT screen as guidance to the operator.

2.3 Measurement Results

An example of measurement results by the camber meter is shown in **Fig. 4**. In Fig. 4 (a), the measured value of distance M and its approximated curve are shown; in Fig. 4 (b), the measured and calculated values of the camber profile are given. In these figures, M and the camber profile have been approximated by the 4th and 6th order polynomials, respectively. Through the use of this camber measuring method, bending shapes having various curvatures which are generated during rolling can be faithfully expressed.

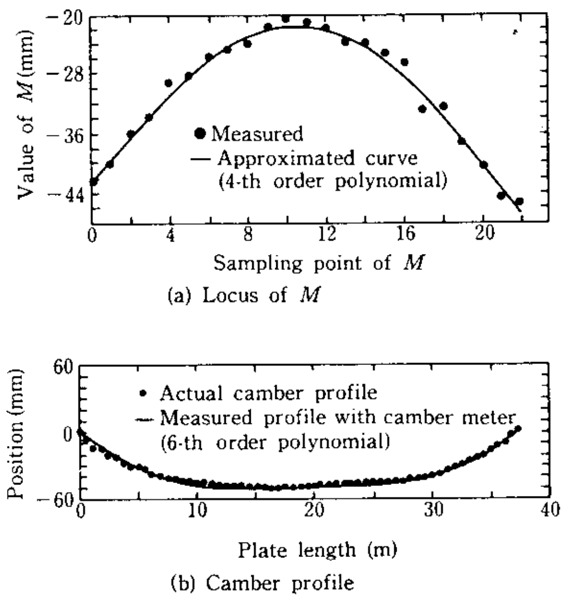


Fig. 4 An example of camber profile measurement

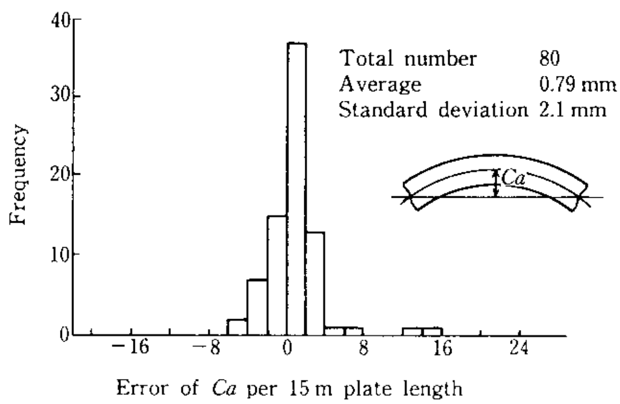


Fig. 5 Histogram of measurement error with camber meter

A histogram of measurement errors with the camber meter is shown in **Fig. 5**, where errors are evaluated by converting them into an error of camber values (Ca in the figure) per 15 m of plate length. This camber meter has attained the accuracy as indicated by an average error value of 0.79 mm and a standard deviation of 2.1 mm.

3 Camber Control

Camber control of the plate is effected on the basis of the result of camber measurement. First, the next-pass aimed wedge (difference in thickness between the right and left edges of the plate), which is necessary for correcting the camber, is calculated from the measurement results of camber. Then, rolling control is effected in the next pass, so that the aimed wedge can be satisfied.

In the following, models necessary for camber control and the detailed control method will be described.

3.1 Models for Camber Control

To control camber, the following two models have been prepared:

(1) Model showing the relation between the camber of the plate, which occurs as a result of rolling, and the wedge: This model is used for calculating, from the camber profile of the plate, the aimed wedge which is necessary for correcting the camber profile.

(2) Model for observing the wedge during rolling: This model is used for controlling the wedge according to the target.

The above models are described in detail below.

3.1.1 Relational expressions between camber and wedge

(1) Basic Model

When no consideration is given to the three-dimensional deformation such as the width-directional metal flow during rolling, the relation between the camber and wedge is inevitably determined by the difference in elongation between the right and left edges of the rolled material, and can be expressed by the following equations:

$$\rho_1 = \frac{1}{\lambda^2}\left(\rho_0 + \frac{\Delta\psi}{B}\right) \quad \cdots\cdots(7)$$

$$\lambda = \frac{h}{H}$$

$$\Delta\psi = \frac{h_{df}}{h} - \frac{H_{df}}{H}$$

where ρ_1: Delivery curvature

Table 2 Rolling conditions in model mill

Specifications of model mill	Type of model mill		2 Hi
	Work roll diameter		200 mmϕ
	Work roll barrel length		500 mm
Conditions of experiment	Material		Pure lead
	Dimensions of specimen	Thickness	2, 4, 8 mm
		Width	150, 250, 350 mm
		Length	500 mm
	Wedge ratio		0, 3, 6, 10%
	Reduction ratio		10, 20%

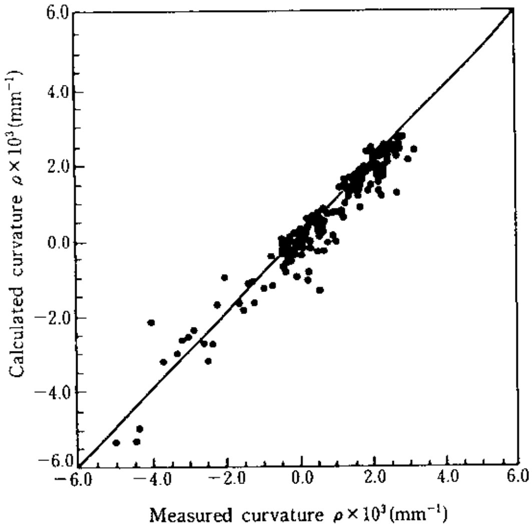


Fig. 6 Comparison of curvature ρ between calculated and measured values (lead rolling)

ρ_0: Entry curvature
λ: Elongation ratio
$\Delta\psi$: Changes in wedge ratio
B: Plate width
h: Delivery thickness
H: Entry thickness
h_{df}: Delivery wedge (Work-side plate thickness – drive-side plate thickness)
H_{df}: Entry wedge (Work-side plate thickness – drive-side plate thickness)

(2) Verification by Model Mill

In commercial rolling, it is considered that the plate is affected by the three-dimensional deformation, and particularly, within a larg plate-thickness range, relaxation takes place so as to lesson the degree of camber.

Therefore, a laboratory scaled experiment using pure lead has first been conducted to find out the relation between the camber and wedge quantitatively. In this experiment, a straight plate without a wedge has been rolled at room temperature with a difference in roll gap between the right and the left hand sides. The specification of the test mill and test conditions are shown in **Table 2**. The thickness of the test plate and the roll diameter of the model mill are equivalent to 1/6 of those in commercial rolling operation.

A comparison between the measured and the calculated values of the plate curvature is shown in **Fig. 6**. The calculated value has been obtained from Eq. (7) using a measured wedge. The measured and the calculated values have shown a good agreement, well corresponding to the results obtained by Nakajima, et al.[3)]

In addition, no influence of plate thickness has been observed, and within the scope of this experiment, the phenomenon of the camber decrease caused by the width-directional metal flow has not been observed.

(3) Verification by Commercial Rolling Mill

Next, a test using commercial rolling mill has been conducted in order to obtain a relation between the camber and the wedge in commercial rolling. In this test, rolling with a difference in roll gap between the right and the left hand sides has been performed at the final pass, thereby intentionally generating the camber. Further, the camber and wedge before and after the final pass have been measured using the camber meter and crown meter, respectively. The test material is a mild steel, measuring 2 000 mm, 3 000 mm, and 4 000 mm in width and 10 mm and 15 mm in thickness.

A comparison of the measured and the calculated values of the plate curvature is shown in **Fig. 7**. The calculated value has been obtained from Eq. (7). Both the measured and the calculated values coincide with each other qualitatively, and no obvious difference due to plate thickness and width has been observed.

In comparison between Figs. 6 and 7, the scatter of the result of commercial mill test is larger than that in the lead model test. The reasons for this may be as follows:

(1) In the lead model test, a wedge was formed relatively large, thus making the resultant camber large enough to absorb the effect of the width-wise metal flow.
(2) In the commercial roll test, the wedge was suppressed to a small value, making the resultant camber small unable to absorb the effect of the widthwise metal flow, thus causing the effect to show.

The above-mentioned problems have been corrected

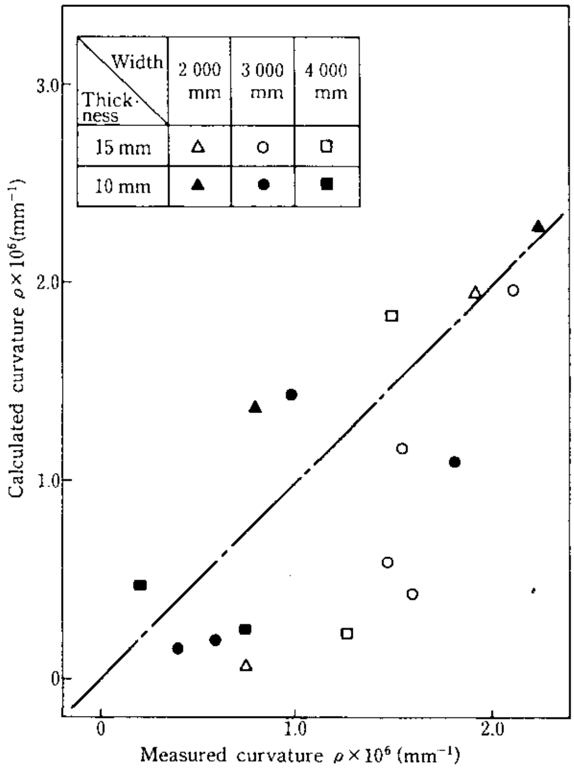


Fig. 7 Comparison of curvature ρ between calculated and measured values (steel rolling)

by adjusting the model equation depending upon the plate width and thickness in the commercial rolling operation.

3.1.2 Wedge observation model

This model is used for observing the wedge during rolling. Since this model is incorporated in the dynamic control system, calculation speed is required and the simplification of the model equation becomes necessary. In forming this model, careful attention was given to maintaining accuracy and also simplifying the model equation.

(1) Basic Model

The deformation condition of rolling mill when roll deformation was disregarded is shown in **Fig. 8**. A basic model, as shown below, is obtained from the dynamical relation between the rolled material and the mill, taking into consideration the sideslipping of the rolled material, difference between the right and the left mill constants of the rolling mill, difference between the right and the left roll gaps, and work roll crown quantity:

$$h_{df} = \left(\frac{P_W}{K_W} - \frac{P_D}{K_D}\right)\frac{B}{l_0} + S_{df}\frac{B}{l_0} + 4C_W \sin\left(\frac{\delta}{W}\pi\right)\sin\left(\frac{B}{2W}\pi\right) \cdots\cdots (8)$$

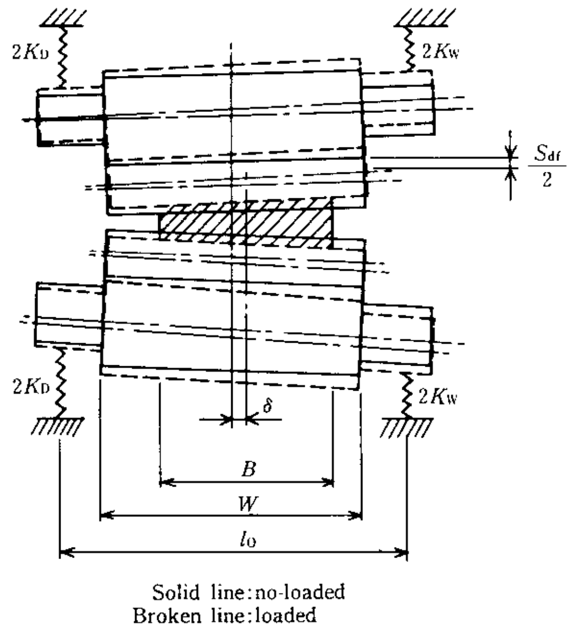


Fig. 8 Wedge model on the assumption that rolls are rigid

where P_W: Work-side rolling load
P_D: Drive-side rolling load
K_W: Work-side mill constant
K_D: Drive-side mill constant
B: Plate width
S_{df}: Difference between right and left roll gaps (Work-side gap − drive-side gap)
C_W: Work roll crown
W: Roll barrel length
l_0: Distance between roll chocks
δ: Side slipping value (Deviation of the central position in the width direction of the plate from the central position of the rolling mill. It is also called the "off-center value". Deviation towards the work side is taken as "positive".)

(2) Simulation by Split Model

Previously the model equation was formed by neglecting roll deformation, but actually, roll deflection, and roll flattening have influence on it, resulting in errors. Therefore, the authors performed simulation by a split model[9] and prepared a correction equation from the difference between the results of the basic model and simulation.

An example of simulation of the relation between the sideslipping, difference between the right and the left roll gaps, and wedge is shown in **Fig. 9**. The wedge has a linear relation with the difference between the right and the left roll gaps. Also, since

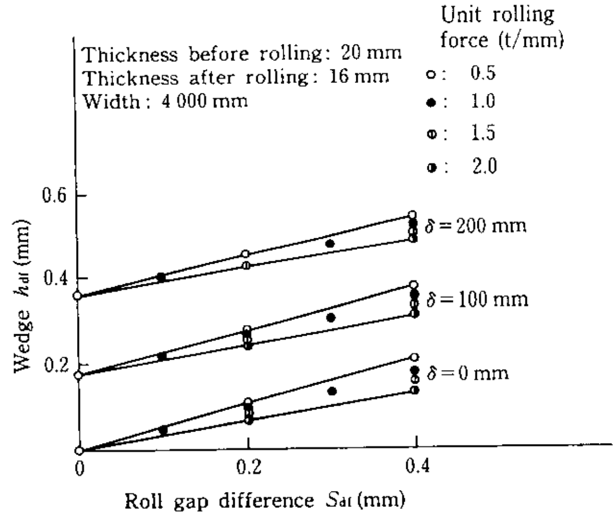


Fig. 9 Relation between wedge h_{df} and roll gap difference

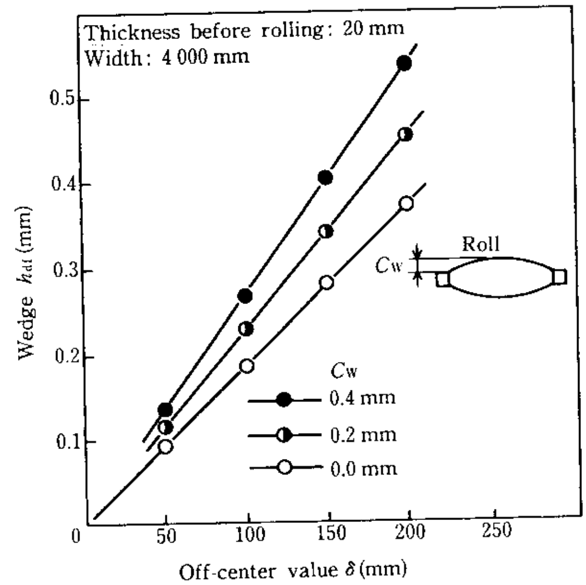


Fig. 10 Influence of work roll crown C_W on wedge h_{df}

the slope of this straight line is constant independent of the sideslipping value, it is known that the effect of the difference between the right and the left roll gaps and the effect of the sideslipping value on the wedge can be treated independently from each other.

An example of the result of simulation is also shown in **Fig. 10**. The effect of the sideslipping value on the wedge varies with the work roll crown. This indicates the validity of the basic model equation (8). On the basis of the above-shown simulation results, the basic model equation (8) is corrected and the following Eq. (9) has been obtained. In this Eq. (9), the effect of entry-side wedge H_{df} is also taken into consideration.

$$h_{df} = (e_1 + e_2(C_W))f_1(\delta) + e_3 f_2(P_{df}) + e_4 f_3(S_{df}) + e_5 f_4(H_{df}) \cdots\cdots\cdots\cdots (9)$$

where P_{df}: Rolling load difference (Work-side load − drive-side load)

$e_1 \sim e_5$: Correction coefficient for basic model equation

$f_1 \sim f_4$: Basic model portion

(3) Verification of Model Equation

The wedge observation model has been verified by commercial mill rolling. The result is shown in **Fig. 11**. The calculated values have been obtained from Eq. (9), and in obtaining the measured values, a micrometer has been used after rolling. Both the test method and test material are the same as those used in the camber generation test. In various rolling conditions and roll chances, calculated values show a good agreement with measured values.

In **Fig. 12**, an example of the calculated and measured values of the wedge in the length direction of the plate is shown. The calculated value by the model equation shows a good correspondence to

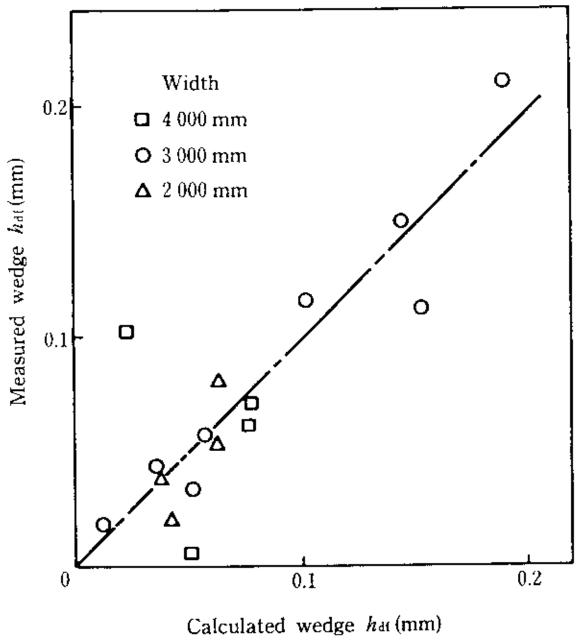


Fig. 11 Comparison of wedge h_{df} between calculated and measured values

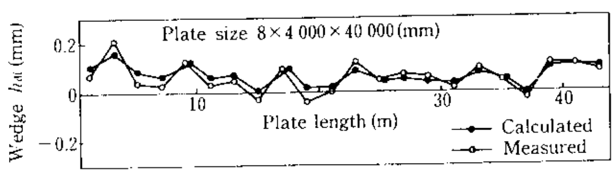


Fig. 12 Comparison of plate wedge between measured and calculated

the measured value, and accuracy in using the model for control purposes has satisfactorily been ensured.

3.2 Camber Control Method

This section concretely describes the method of controlling the camber after the measurement.

First, the measured camber $y = F(x)$, which has been described by the nth order polynomial, is converted into curvature ρ by the following equation:

$$\rho(x) = \frac{F''(x)}{(1 + F'(x)^2)^{3/2}} \quad \cdots\cdots\cdots\cdots\cdots(10)$$

where

$$F'(x) = \frac{dF(x)}{dx}$$

$$F''(x) = \frac{d^2F(x)}{dx^2}$$

Equation (10) expresses a curvature at an arbitrary point x in the length direction of the plate.

Next, the aimed wedge $h_{df}^*(x)$ of the succeeding pass is calculated to correct the camber of curvature ρ.

$$h_{df}^*(x) = \left(\frac{H_{df}(x)}{H} - B\rho(x)\right)h \quad \cdots\cdots\cdots(11)$$

where $h_{df}^*(x)$: Aimed wedge at position x in the length direction of plate

H_{df}: Current wedge at position x in the length direction of plate

h: Delivery-side aimed plate thickness

Equation (11) can be obtained by equating the delivery side camber ρ_1 to zero in Eq. (7). In the actual control process, the plate is sub-divided in the length direction, and aimed wedges of all these sections are calculated.

When the succeeding pass begins, the feedback control system shown in **Fig. 13** is constructed and wedge control is effected to generate the aimed wedge. At this time, the following equation is used for observing the wedge:

$$h_{df} = \alpha_1 P_{df} + \alpha_2 S_{df} + \alpha_3 \delta + \alpha_4 \quad \cdots\cdots\cdots(12)$$

where $\alpha_1 \sim \alpha_4$: coefficients

Wedges are observed using this Eq. (12), and the screwdown position is adjusted depending upon the deviation from the aimed wedge. Equation (12) is a simplified form of Eq. (9).

If the wedge is regulated to the aimed wedge, the plate after the pass will acquire a camberless profile.

3.3 Application of Camber Control to Commercial Rolling

3.3.1 Actual process of camber control

Camber is measured in the direction of arrow (A) in Fig. 2, and controlled in the direction of arrow (B).

Timing of the camber control is shown in **Fig. 14**. In the case of the frontside-finishing, the final pass becomes the control pass, and in the case of the backside finishing, one pass before the last becomes the control pass.

After camber measurement, the aimed wedge is calculated by the process computer on the basis of Eqs. (10) and (11), and the result is transmitted to the lower-rank microcomputer. When the control pass begins, the wedge control shown in Fig. 13 is executed.

Instruction value of the roll gap difference between the right side and the left, S_{df}^*, and aimed wedge h_{dt}^* are provided with limit values to prevent abnormal control. Also for the controller, the PI (proportion and integral) control functions are used.

At present, the plate mill has no sideslip detecting sensor and the sideslipping value cannot be measured. Therefore, an additional model is formulated which

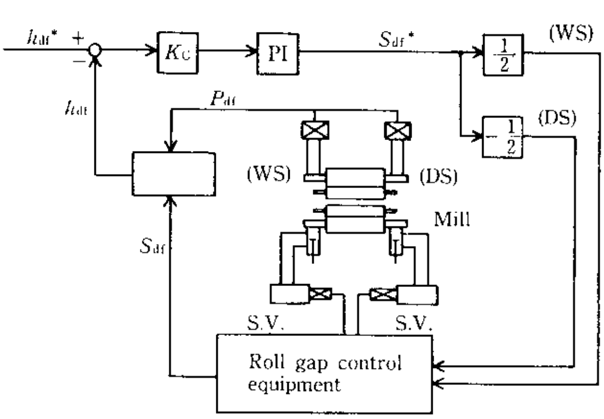


Fig. 13 Block diagram of camber control

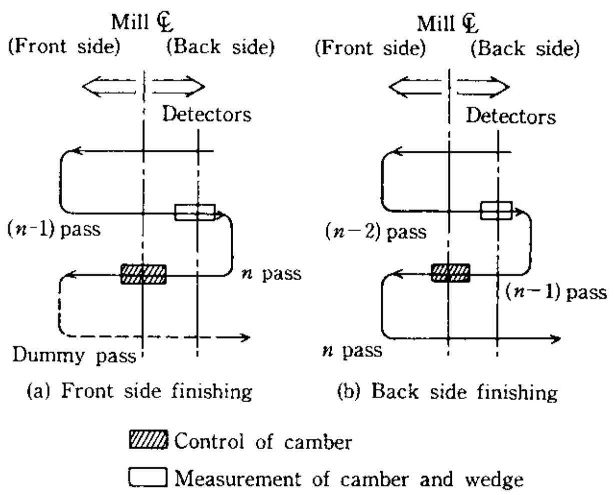


Fig. 14 The timing of camber control

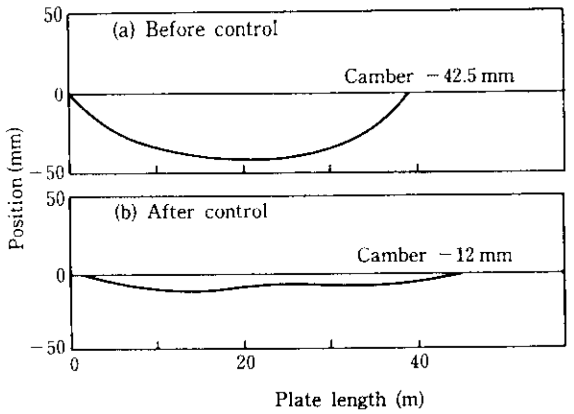


Fig. 15 An example of camber control

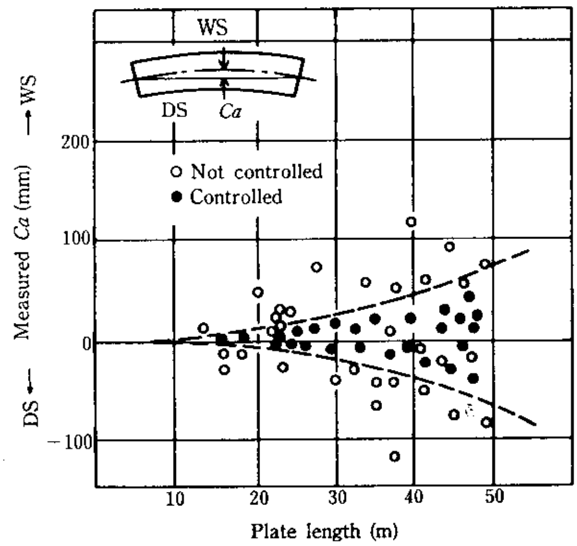


Fig. 16 Effect of camber control

expresses the rolling load difference which is caused by sideslipping, and a wedge observation model is used in which the term of the sideslipping value is eliminated from Eq. (12).

3.3.2 Control Results

An example of control effected on a 15-mm-thick plate is shown in **Fig. 15**. The camber which amounted to 42.5 mm at the time of measurement has been corrected down to 12 mm by camber control, thereby clearly indicating the effectiveness of the camber control.

Effectiveness of camber control execution in the commercial rolling process is shown in **Fig. 16**. This is the rolling result at the same roll chance. When camber control is effected, the scatter of the results has been reduced to half, indicating the effectiveness of camber control. Camber has been suppressed within 5 mm per 15 mm of the plate length.

4 Conclusions

The Plate Mill at Mizushima Works has developed its unique camber measuring and controlling methods and realized their applications to commercial operations. The outstanding features of the camber meter and the camber control are summarized below.

(1) Camber can be measured at a location very near to the mill site without being adversely affected by sideslipping and turning of the plate during rolling.

(2) Through expressing the plate camber by means of the nth order polynomial, the actual camber profile can be faithfully expressed and the measured camber profile can be immediately reflected on the control.

(3) From the measured result of the camber, the aimed wedge is calculated, and feedback control is effected to the wedge of the succeeding pass, thereby correcting the camber of the succeeding pass.

The above-mentioned system is now smoothly operating, and is effective in reducing the camber, contributing to an improvement in yield, reduction in poor grades, and enhancement of efficiency in the shearing process.

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